Design of experiment (DOE)

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DOE Tell us…

Is output changed? , while input variables are changed.

Only this ! not else.

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Terminology

Factor : Independent variable of experiment.

Symbol by : Capital character (A, B, …..)

Level : Number of factor changing

Symbol by :Character (a, b, …..)

Treatment : Value of each changed-independent variables.

Response : Output from system that independent variables are changed.

Replication : Number of run in one treatment (n).

Randomization : Do something by un-specify parameter.

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1 Factor, 4 Levels, 6 Replication (paper manufacturing process)

A is density of wood material that effect tensile strength of paper.

Wood density (%)	Tensile strength of paper (Response value)						Sum	Avg.
	1	2	3	4	5	6		
5	7	8	15	11	9	10	60	10.00
10	12	17	13	18	19	15	94	15.67
15	14	18	19	17	16	18	102	17.00
20	19	25	22	23	18	20	127	17.83
							318	15.96

Levels = 4

Replicate = 6

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2 Factors, 2 levels and 3 levels, 3 Replications
(Car painting process)

Factor 1 : 3 types of color

Factor 2 : 2 types of coating process

Number of Replication = 3

Factor 2 = 2 levels

Factor 1 =3 levels

Color type	Coating method		Sum
	Brush	Spray	
1	4.0, 4.5 , 4.3	5.4, 4.9 , 5.6	28.7
2	5.6, 4.9, 5.4	5.8, 6.1, 6.3	34.1
3	3.8, 3.7, 4.0	5.5, 5.0, 5.0	27.0
Sum	40.2	49.6	89.8

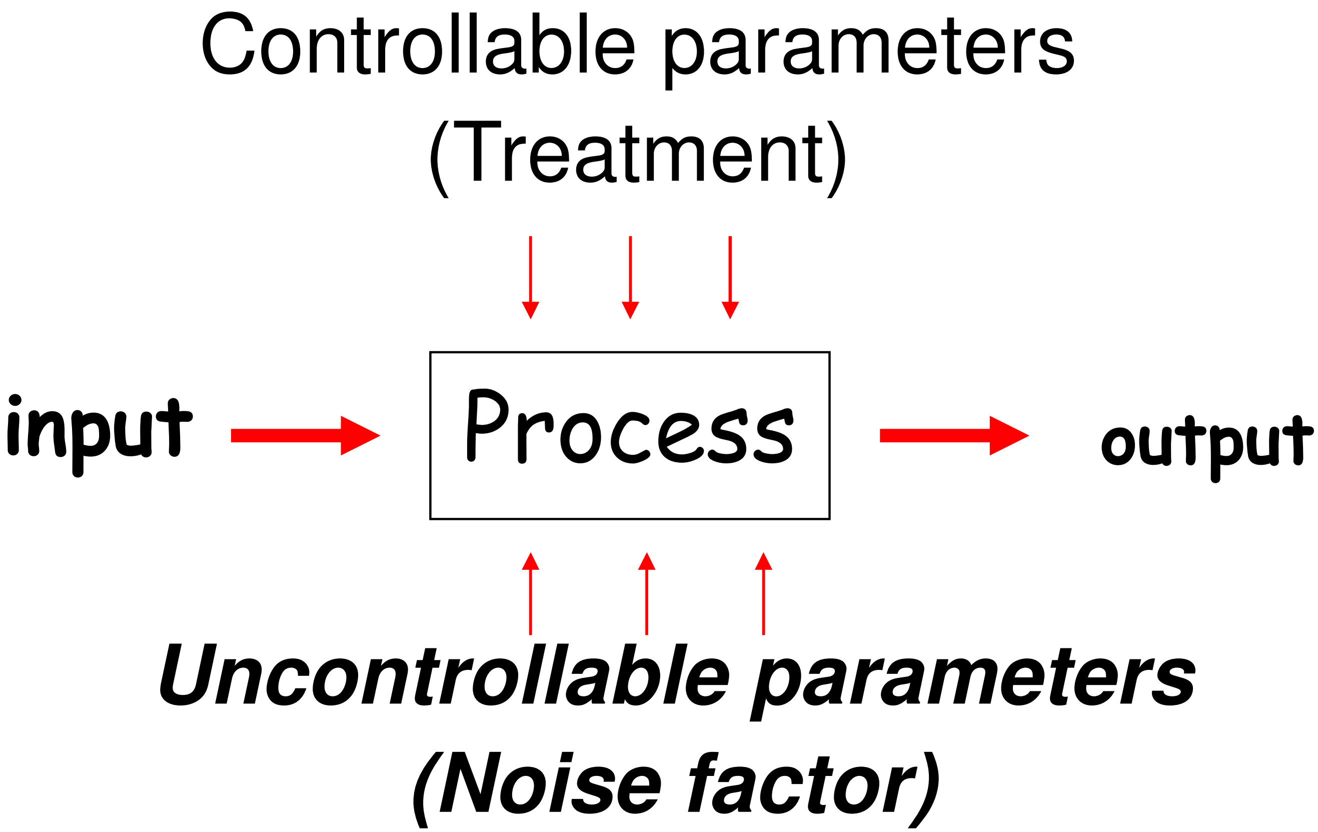
First

Second

third Treatment

Responses

Noise factors



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Factor effect tensile strength of paper

Controllable Factors

1. Wood density
2. % of glue ingredient.
3. Drying process

Uncontrollable Factors

1. Wood moisture
2. PH of water input to process
3. Accuracy of measure equipment

Vary 1 or 2 controllable by n levels.
Fix others controllable.

DOE will find effect of only controllable to response value

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ANalysis Of Variance :ANOVA

$\mathbf{Y_{ij}}$ is Response variable of experiment : Level i and Replication j

μ is average

τ_i is Factor effect in level i

e_{ij} is Random error

$$\mathbf{Y_{ij}} = \mu + \tau_i + e_{ij} \qquad \begin{cases} i = 1,2,3,\ldots,a \\ j = 1,2,3,\ldots,n \end{cases}$$

One- way analysis of variance model

2 assumptions

1. Random error is normal and independent distribution for each experiment, by average = 0 and variance = s^2
2. s^2 is as same every treatment

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Symbol

Level	Response	Sum	Avg.
1	Y_{11} Y_{12} .... Y_{1n}	$Y_{1.}$	$\bar{Y}_{1.}$
2	Y_{21} Y_{22} .... Y_{2n}	$Y_{2.}$	$\bar{Y}_{2.}$
•	. . ..	.	.
•	. . ..	.	.
•	. . ..	.	.
a	Y_{a1} Y_{a2} .... Y_{an}	$Y_{a.}$	$\bar{Y}_{a.}$
		$Y_{..}$	$\bar{Y}_{..}$

Symbol

$$Y_{i.} = \sum_{j=1}^{n} Y_{ij} \qquad \bar{Y}_{i.} = Y_{i.}/n$$

$$Y_{..} = \sum_{i=1}^{a} \sum_{j=1}^{n} Y_{ij} \qquad \bar{Y}_{..} = Y_{..}/n$$

i = 1,2 , ... , a (Number of level)

j = 1, 2, ... , n (Number of Replication)

N = an (Number of Observation)

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ANOVA 1 factor

1 : Create Hypothesis

$$\mathbf{H_0} : \tau_1 = \tau_2 = \ldots\ldots \tau_a = 0$$

$$\mathbf{H_1} : \tau_i \neq 0 \text{ at lest 1}$$

2 : Analysis

"ANOVA" separates noise effect from treatment factor effect

And then test only treatment factor effect

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ANOVA Calculation

$$SS_{total} = SS_{factor} + SS_{error}$$

$$SS_{total} = \sum_{i=1}^{a} \sum_{j=1}^{n} (\bar{Y}_{ij} - \bar{Y}_{..})^2$$

$$= n \sum_{i=1}^{a} (\bar{Y}_{i.} - \bar{Y}_{..})^2 + \sum_{i=1}^{a} \sum_{j=1}^{n} (\bar{Y}_{ij} - \bar{Y}_{i.})^2$$

$$SS_{factor} = \sum_{i=1}^{a} \frac{Y_{i.}^2}{n} - \frac{(Y_{..})^2}{an}$$

$$SS_{error} = SS_{total} - SS_{factor}$$

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ANOVA : find Mean Square(MS)

$$MS = \frac{SS}{\text{degree of freedom}}$$

Data are collected from sample

degree of freedom (df) = Number of sample of each value -1

$df_{factor} = a-1$

$df_{total} = N-1 = an - 1$

$df_{error} = a(n-1) = an-a = N-a$

$$MS_{factor} = \frac{SS_{factor}}{a-1}$$

$$MS_{error} = \frac{SS_{error}}{a(n-1)}$$

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ANOVA : F- test

$$F_o = \frac{MS_{factor}}{MS_{error}}$$

If $F_o > F_{\alpha,(a-1),a(n-1)}$
reject H_0

If $F_o < F_{\alpha,(a-1),a(n-1)}$
Not reject H_0

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ANOVA Table:

Source of variance	SS	df	MS	F_o
Between treatment (Factor effect error)	SS_{factor}	a-1	MS_{factor}	$F_o = \frac{MS_{factor}}{MS_{error}}$
Inner treatment (Random error)	SS_{error}	a(n-1)	MS_{error}	
Total error	SS_{total}	an-1		

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Example One-way ANOVA
(Tensile strength of paper)

$$SS_{total} = \sum_{i=1}^{4} \sum_{j=1}^{6} Y_{ij}^{2} - (Y..^{2})/\ n$$

$$= (7)^2+(8)^2+ \ . \ . \ . \ +(20)^2 \ -(383)^2/24 \qquad = 512.96$$

$$SS_{factor} = \sum_{i=1}^{4} (Y_i.^{2})/n \ - \ (Y..)^2/an$$

$$= [(60)^2+(94)^2+(102)^2+(127)^2]/6 - (383)^2/24 \quad = 382.79$$

$$SS_{error} = SS_{total} - SS_{factor}$$

$$= 512.96 - 382.79 \qquad = 130.17$$

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Example in ANOVA table

Source of variance	SS	df	MS	F_o
Wood density	382.79	3	127.60	19.61
Random error	130.17	20	6.51	
Total error	512.96	23		

$F_{0.01,3,20}$ = 4.94 If $F_o > F_{0.01,3,20}$: will Reject H_0

Compare 19.61 (F_0) > 4.94 ($F_{0.01,3,20}$)

Reject H_o

Wood density effect Tensile Strength of Paper

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