Statistical Concept and Hypothesis Testing in Simulation

Lecture 6-7

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From Simulation Process

Input Data Side

Real System

↓

Collect Data

↓

Distribute Data
for
Input to Model

Output Data Side

Tally Statistic Data

↓

Distributed Data

↓

Testing Process &
Output Analysis

A lot of Data be accommodated.
Are they correctly done?

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Data are transformed.

Input Data Side

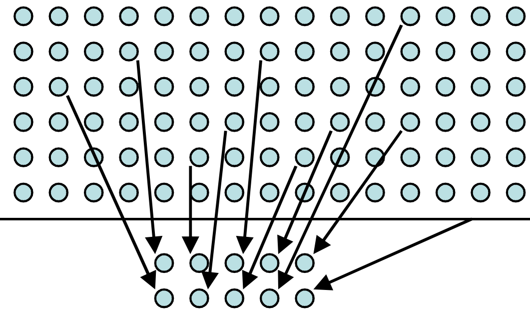
Real System

↓

Collect Data

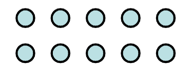
↓

Distribute Data
for
Input to Model



Normal (μ,σ),etc.

Output Data Side



Normal (μ,σ), etc.

Test for Mean
Test for Variance
Test for Ratio
Test for Anything….

Tally Statistic Data

↓

Distributed Data

↓

Testing Process &
Output Analysis

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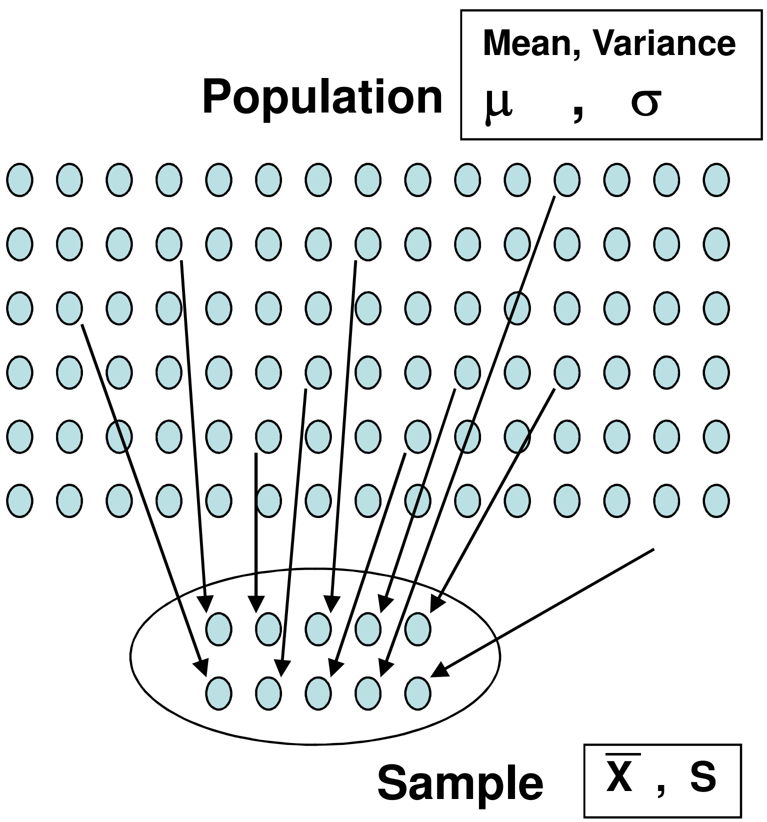
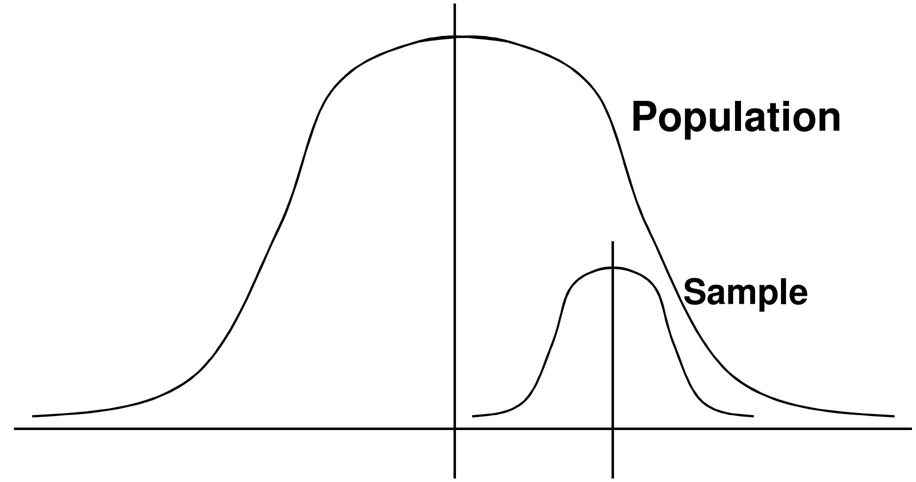
Related Statistical Methods

Fit Distribution
Hypothesis Testing
Design of Experiment
Sample Size

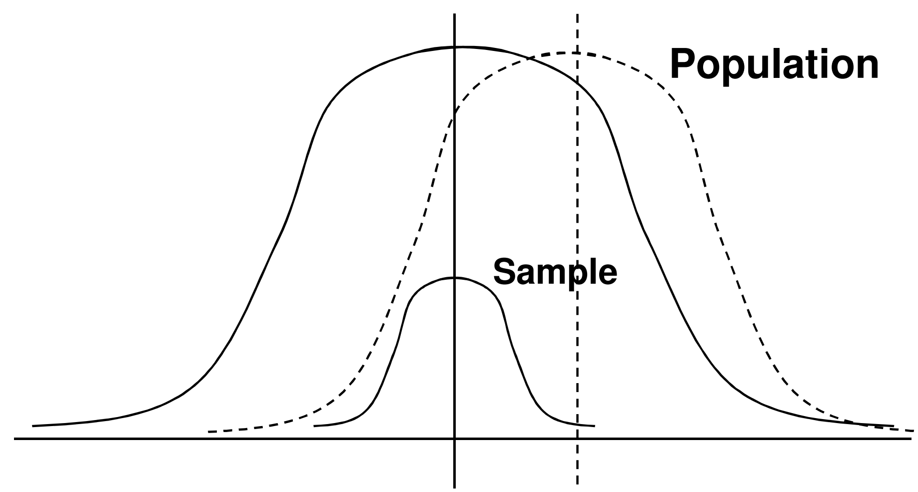
Hypothesis Testing

Principle Concept





Is sample represented population?



Or represented another population?

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Imagination

Actual mean of all students weight = 55 kgs.
Sampling meets x-bar = 57 kgs.
Experimentor thinks = 58 kgs.

Current output rate = 200 pcs./hr.
After alternative simulation run 50 times
average output(x-bar) = 185 pcs./hr.
Experimentor thinks = 185 pcs./hr. ; Decrease
or thinks = 200 pcs./hr. ; No change

How to get correctly conclusion.

The Hypothesis

$\overline{X} = \mu_0$: Null Hypothesis

$\overline{X} \neq \mu_0$: Alternative Hypothesis

This is 2 tails testing (for normal distribution)

Others are :

0ne tail testing (for normal distribution)

Chi-square testing

Binomial testing

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Type of Errors from Statistical Conclusion

	(H_0) Null hypothesis is	
	True	False
Not reject Null	√	β(Type 2 error)
Reject Null	α(Type I error)	√

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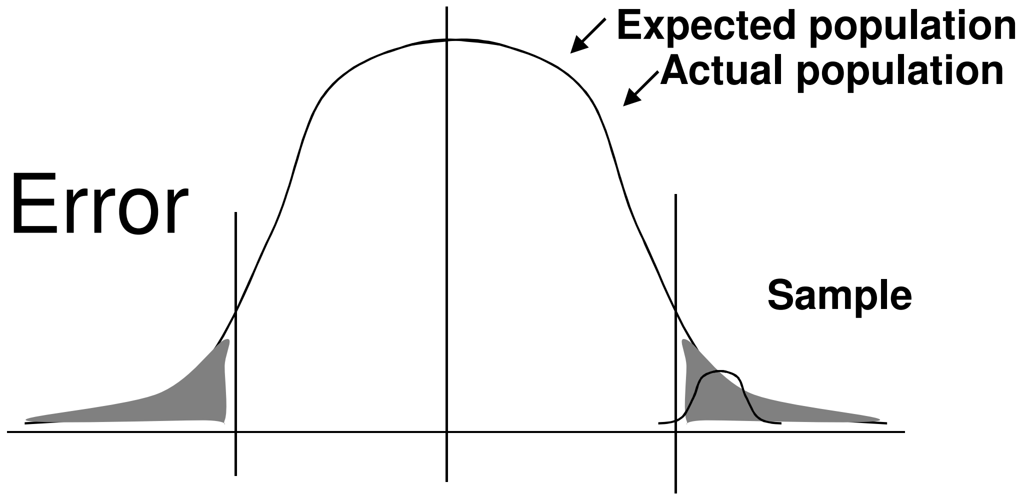
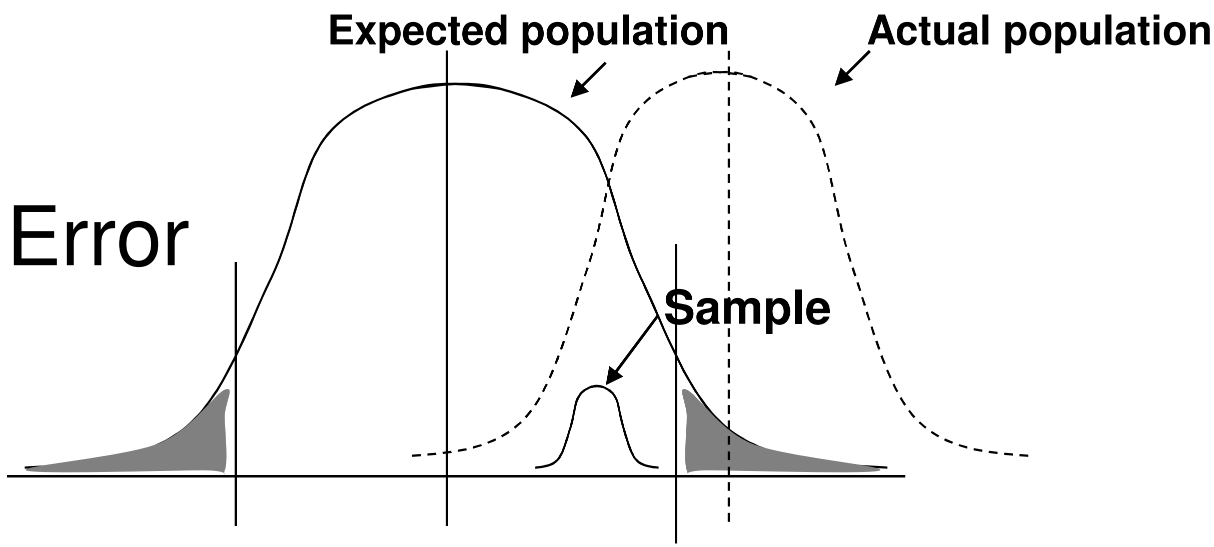
Graphical Explanation of Errors

Type 1 Error



Type 2 Error



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Testing Process

1) Construct Hypothesis : $\overline{X} = \mu_0$:H $_0$
   $\overline{X} \neq \mu_0$:H $_1$
2) Define Power of test (Confidential Level)
   95% or 0.95 is normal recommendation

3) Calculate Parameter
   Z, t, F, λ,etc.
4) Compare and Conclusion

types of Hypothesis Testing

1) Value testing by σ is known

2) Value testing by σ is unknown

3) Variance testing

4) Binomial distribution (proportion) testing

1.1) Single value testing by σ is known

1- Create hypothesis

$\overline{X} = \mu_0$:H_0

$\overline{X} \neq \mu_0$:H_1

2- Define power of test = 0.95

collect data = n
calculate x-bar

3- Calculate parameter

$$Z_0 = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

4- Reject null hypothesis while $\left|Z_0\right| > Z_{\alpha/2}$

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1.2) Compare 2 populations by known $\sigma_1^2 = \sigma_2^2$

1- Create hypothesis

$\mu_1 = \mu_2 : H_0$

$\mu_1 \neq \mu_2 : H_1$

2- Define power of test = 0.95

collect data = n
calculate x-bar

3- Calculate parameter

$$Z_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

4- Reject null hypothesis while $|Z_0| > Z_{\alpha/2}$

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2.1) Single value testing by σ is unknown

1- Create hypothesis $\bar{X} = \mu_0$:H_0

$\bar{X} \neq \mu_0$:H_1

2- Define power of test = 0.95

collect data = n
calculate x-bar
calculate S

3- Calculate parameter

$$t_0 = \frac{\bar{x} - \mu_0}{S / \sqrt{n}}$$

4- Reject Hypothesis while $|t_0| > t_{\alpha/2, n-1}$

2.2) Compare 2 populations by σ_1, σ_2 is unknown but known $\sigma_1 = \sigma_2$

1- Create hypothesis

$\mu_1 = \mu_2 : H_0$

$\mu_1 \neq \mu_2 : H_1$

2- Define power of test = 0.95

collect data = n

calculate x-bar

calculate S

3- Calculate parameter

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

4- Reject null hypothesis while $|t_0| > t_{\alpha/2, n_1 + n_2 - 2}$

2.3) Compare 2 populations by σ_1, σ_2 is unknown but know $\sigma_1 \neq \sigma_2$

1- Create hypothesis

$\mu_1 = \mu_2 : H_0$

$\mu_1 \neq \mu_2 : H_1$

2- Define power of test = 0.95

collect data = n

calculate x-bar

calculate S

3- Calculate parameter

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

$$t_0 = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

4- Reject null hypothesis while $|t_0| > t_{\alpha/2,\nu}$

$$\nu = \left[\frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)}{\frac{(s_1^2/n_1)^2}{n_1+1} + \frac{(s_2^2/n_2)^2}{n_2+1}} \right] - 2$$

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3.1) Single variance testing

1- Create hypothesis

$S^2 = \sigma^2 : H_0$

$S^2 \neq \sigma^2 : H_1$

2- Define power of test = 0.95

collect data = n

calculate x-bar

3- Calculate parameter

$$\chi_0^{\ 2} = \frac{(n-1)s^{\ 2}}{\sigma_0^{\ 2}}$$

4- Reject Hypothesis while

$$\chi_0^{\ 2} > \chi^2_{\ /2,n-1}$$

3.2) Compare 2 variances

1- Create hypothesis

$S_1^2 = S_2^2 : H_0$

$S_1^2 \neq S_2^2 : H_1$

2- Define power of test = 0.95

collect data = n
calculate x-bar

3- Calculate parameter

$$F_0 = \frac{s_1^2}{s_2^2}$$

4- Reject Hypothesis while $F_0 < F_{1-\alpha/2, n_1-1, n_2-1}$

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4.1) Single binomial testing

1- Create hypothesis

$P = P_0 : H_0$
$P \neq P_0 : H_1$

P is Probability of happen of interest event

Example : Run n batchs and interest batch of average part in queue <x

2- Define power of test = 0.95

collect data = n
calculate x-bar

3- Calculate parameter

$$Z_0 = \frac{(x + 0.5) - np_0}{\sqrt{np_0(1 - p_0)}} ; x < np_0$$

$$\frac{(x - 0.5) - np_0}{\sqrt{np_0(1 - p_0)}} ; x > np_0$$

4- Reject Hypothesis while $|Z_0| > Z_{\alpha/2}$

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4.2) Compare 2 binomial distributions

1- Create hypothesis

$P_1 = P_2 : H_0$
$P_1 \neq P_2 : H_1$

P_1,P_2 is Probability of happen of interest event of population 1 and 2
Example : Run n batchs and interest batch of average part in queue <x

2- Define power of test = 0.95

collect data = n
calculate x-bar

3- Calculate parameter

$P_1 = x_1/n_1$

$P_2 = x_2/n_2$

$P = (n_1P_1 + n_2P_2) / (n_1 + n_2)$

$$Z_0 = (P_1 - P_2) / \sqrt{P(1-P)(1/n_1 + 1/n_2)}$$

4- Reject Hypothesis while $\left|Z_0\right| > Z_{\alpha/2}$

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